Quiz 8 MATH 3280 3.00 Nov 25, 2020

Given name and surname:_____

Student No:_____

Signature:_____

INSTRUCTIONS:

1. Please write everything in **ink**.
2. This quiz is a 'closed book' test, duration **25** minutes.
3. Only non-programmable calculators are permitted.
4. The text has two pages, and it contains two questions. Read the question carefully. Fill in answers in designated spaces. Your work must justify the answer you give. Answers without supporting work will **not** be given credit.

GOOD LUCK!

Question 1 A square matrix $Q \in Mat_{m\times m},\ m \in \mathbb{N}$ is called stochastic matrix if its every element is a probability, and the sum of all elements in each row is one. Prove that if the matrix Q is stochastic, then the matrix $Q^k,\ k \in \mathbb{N}$ is also stochastic.

Question 2 Suppose that a non-life insurer classifies its motor insurance policyholders according to two states, preferred (#1) and standard (#2), at time zero and reclassification occurs at the beginning of each policy year. You are given the following k-th year non-homogeneous transition probability matrix

$$Q_k = \begin{pmatrix} 0.65 & 0.35 \\ 0.50 & 0.50 \end{pmatrix} + \frac{1}{k+1}\begin{pmatrix} 0.15 & -0.15 \\ -0.20 & 0.20 \end{pmatrix}$$

Given that an insured is preferred at the start of the second year:

- Find the probability that the insured is still preferred at the start of the third year.
- Find the probability that the insured transitions from being preferred at the start of the third year to being standard at the start of the fourth year.

Q.1

$Q^k \cdot \underline{1} = \underbrace{(Q \cdot \ldots \cdot Q)}_{k \text{ times}} \cdot \underline{1} = \underbrace{Q \cdot \ldots \cdot Q}_{(k-1) \text{ times}} \cdot (Q \cdot \underline{1}) = Q \cdot \ldots \cdot Q \cdot \underline{1}$

$= \underbrace{Q \cdot \ldots \cdot Q}_{(k-2) \text{ times}} \cdot (Q \cdot \underline{1}) = \underbrace{Q \cdot \ldots \cdot Q}_{(k-2) \text{ times}} \cdot Q \cdot \underline{1} = \ldots = Q \cdot \underline{1} = \underline{1}$,

where $\underline{1} = (1, 1, \ldots, 1)'$ is a k-dimensional vector of ones.

Q.2

$P(Y(2) = 1 \mid Y(1) = 1) = (Q_1)_{1,1} = 0.725$

$P(Y(3) = 2 \mid Y(2) = 1) = (Q_2)_{1,2} = 0.3$

The End.