Quiz 7 MATH 3280 3.00 Nov 18, 2020

Given name and surname:____________________

Student No:____________________

Signature:____________________

INSTRUCTIONS:

1. Please write everything in **ink**.
2. This quiz is a 'closed book' test, duration **25** minutes.
3. Only non-programmable calculators are permitted.
4. The text has two pages, and it contains two questions. Read the question carefully. Fill in answers in designated spaces. Your work must justify the answer you give. Answers without supporting work will **not** be given credit.

GOOD LUCK!

Question 1 Verify each one of the following expressions and indicate which ones require the assumption of independence of the future life-time random variables $T(x)$ and $T(y)$, where $x, y \geq 0$.

a. ${}_tq_{\overline{x:y}} = {}_tq_x + {}_tq_y - {}_tq_{x:y},\ t \geq 0.$

b. ${}_tp_{\overline{x:y}}\mu_{\overline{x:y}}(t) = {}_tq_x \times {}_tp_y\mu_y(t) + {}_tq_y \times {}_tp_x\mu_x(t),\ t \geq 0.$

c. ${}_{t|}q_{\overline{x:y}} = {}_tq_x \times {}_{t|}q_y + {}_tq_y \times {}_{t|}q_x + {}_{t|}q_y \times {}_{t|}q_x,\ t \geq 0.$

Question 2 You are given that $\mu^{(1)}(t) = 1.8/(9-t),\ t \in (0,\ 9)$ and $\mu^{(2)}(t) = 1.5/(9-t),\ t \in (0,\ 9)$ are the forces of mortality of two life-statuses, say $(0)^{(1)}$ and $(0)^{(2)}$. Assume that these life-statuses have independent future life-times and find the probability that $(0)^{(1)}$ dies first.

Q.1

a. is always true. (min/max).

b. is true under independance

c. is true under independance

Q.2

$$\mathbb{P}\left(T(0)^{(1)} < T(0)^{(2)}\right) = \int_0^9 \mathbb{P}\left(T(0)^{(2)} > t\right) f_{T(0)^{(1)}}(t)\,dt,$$

where

$$\mathbb{P}\left(T(0)^{(2)} > t\right) = \exp\left\{-\int_0^t \frac{1.5}{(9-s)}\,ds\right\} = \exp\left\{\int_0^t 1.5\,d\ln(9-s)\right\}$$

$$= \exp\left\{1.5\ln\left(\frac{9-t}{9}\right)\right\} = \left(\frac{9-t}{9}\right)^{1.5},\quad t \leq 9.$$

So

$$\mathbb{P}\left(T(0)^{(1)} < T(0)^{(2)}\right) = \int_0^9 \left(\frac{9-t}{9}\right)^{1.5} \cdot \left(\frac{9-t}{9}\right)^{1.8} \left(\frac{1.8}{9-t}\right) dt$$

$$= \frac{1.8}{9}\int_0^9 \left(\frac{9-t}{9}\right)^{2.3} dt = 0.2\left[\int_0^9 d\left(\frac{9-t}{9}\right)^{3.3}\right]\frac{1}{3.3}(-9)$$

$$= \frac{4.5}{3.3}\left[-\left(0-1\right)\right] \approx 0.54545.$$