Quiz 5 MATH 3280 3.00 Nov 4, 2020

Given name and surname: ______________________________

Student No: ______________________________

Signature: ______________________________

**INSTRUCTIONS:**

1. Please write everything in **ink**.
2. This quiz is a 'closed book' test, duration **15** minutes.
3. Only non-programmable calculators are permitted.
4. The text has two pages, and it contains two questions. Read the question carefully. Fill in answers in designated spaces. Your work must justify the answer you give. Answers without supporting work will **not** be given credit.

**GOOD LUCK!**

Question 1 Let $\mu(x+t),\ t\in(0,\ 1)$ denote the force of mortality. Assume that this force of mortality changes to $\mu(x+t)-c,\ t\in(0,\ 1),\ c>0$. Find the value of $c$ for which the probability that $(x)$ dies within a year will be quartered.

Question 2 Within a multiple decrement model, the joint distribution of the RVs $T := T(x) \geq 0$ and $I \in \{1.\ 2\}$ is specified by

$$\mu_x^{(1)}(t) = \frac{\theta t^{\alpha-1}e^{-\beta t}}{\int_t^\infty s^{\alpha-1}e^{-\beta s}ds}$$

and

$$\mu_x^{(2)}(t) = \frac{(1-\theta)t^{\alpha-1}e^{-\beta t}}{\int_t^\infty s^{\alpha-1}e^{-\beta s}ds},$$

where $t,\alpha,\beta>0$ and $\theta\in(0,\ 1)$. Obtain expressions for $f_{T,I}(t,i),\ f_I(i)$ and $f_T(t)$ for $t>0$ and $i\in\{1,\ 2\}$. Check whether or not the RVs $T$ and $I$ are independent.

Q. 1

We are looking for $c>0$ in $\tilde{\mu}_x(t) = \mu_x(t) - c$, $x \geq 0$, $t>0$ such that $\tilde{q}_x = \frac{1}{4} q_x$. So

$$\tilde{q}_x = 1-\tilde{p}_x = 1-\exp\left\{-\int_0^1 \tilde{\mu}_x(s)\,ds\right\} = 1-\exp\left\{-\int_0^1(\mu_x(s)-c)\,ds\right\}$$

$$= 1-e^c\, p_x = \frac{1}{4} q_x$$

From here: $1-\frac{1}{4}q_x = e^c p_x$

$$e^c = \frac{1-0.25\,q_x}{1-q_x}$$

$$c = \ln\left(\frac{1-0.25\,q_x}{1-q_x}\right) = \ln(1-0.25q_x) - \ln(1-q_x).$$

Q 2

We have $\mu_x^{(\tau)}(t) = \dfrac{t^{\alpha-1}e^{-\beta t}}{\int_t^\infty t^{\alpha-1}e^{-\beta t}\,dt}$, hence the ratios $\dfrac{\mu_x^{(1)}(t)}{\mu_x^{(\tau)}(t)}$ and $\dfrac{\mu_x^{(2)}(t)}{\mu_x^{(\tau)}(t)}$ are constant in $t>0$ $\Rightarrow$ RVs $T$ & $I$ are $\perp\!\!\!\perp$. Also

$$\mathbb{P}(I=1) = \theta = \frac{\mu_x^{(1)}(t)}{\mu_x^{(\tau)}(t)}$$

The End.

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where $t, \alpha, \beta > 0$ and $\theta \in (0,\ 1)$. Obtain expressions for $f_{T,I}(t,i),\ f_I(i)$ and $f_T(t)$ for $t > 0$ and $i \in \{1,\ 2\}$. Check whether or not the RVs $T$ and $I$ are independent.

and

$$\mathbb{P}(I=2) = 1-\theta = \frac{\mu_x^{(2)}(t)}{\mu_x^{(\tau)}(t)}$$

Further...

$$_tp_x^{(\tau)} = \exp\left\{ - \int_0^t \frac{s^{\alpha-1} e^{-\beta s}}{\int_s^\infty u^{\alpha-1} e^{-\beta u} du} ds \right\}$$

$$= \exp\left\{ \int_0^t d \ln \int_s^\infty u^{\alpha-1} e^{-\beta u} du \right\}$$

$$= \exp\left\{ \ln \int_s^\infty u^{\alpha-1} e^{-\beta u} du \Big|_0^t \right\}$$

$$= \frac{\int_t^\infty u^{\alpha-1} e^{-\beta u} du}{\int_0^\infty u^{\alpha-1} e^{-\beta u} du} = \frac{\beta^{-\alpha}}{\Gamma(\alpha)} \int_t^\infty u^{\alpha-1} e^{-\beta u} du$$

And

$$f_T(t) = \beta^{-\alpha} t^{\alpha-1} e^{-\beta t} / \Gamma(\alpha), \quad t > 0.$$

The End.