

Given name and surname: _____

Student No: _____

Signature: _____

INSTRUCTIONS:

1. Please write everything in **ink**.
2. This quiz is a 'closed book' test, duration **15** minutes.
3. Only non-programmable calculators are permitted.
4. The text has two pages, and it contains two questions. Read the question carefully. Fill in answers in designated spaces. Your work must justify the answer you give. Answers without supporting work will **not** be given credit.

GOOD LUCK!

Question 1 Prove that for two RVs $T(x) \geq 0$, $T(y) \geq 0$, we have

$$\text{Cov}(T(x), T(y)) = \int_0^\infty \int_0^\infty (\mathbb{P}(T(x) \geq u, T(y) \geq v) - \mathbb{P}(T(x) \geq u) \mathbb{P}(T(y) \geq v)) \, du \, dv,$$

given that the covariance is well-defined and finite. Use this identity to show that if the RVs $T(x) \geq 0$, $T(y) \geq 0$ are PQD, that is positively quadrant dependent, then these RVs are positively correlated.

Question 2 Let $T'(x)$, $T'(y)$, ξ be independent RVs distributed gamma with shape parameters γ'_x , γ'_y , γ , respectively, and rate parameter λ ; all parameters are positive. Also, set $T(x) = T'(x) + \xi$ and $T(y) = T'(y) + \xi$. Show that the RVs $T(x)$ and $T(y)$ are PQD. Derive the covariance of the RVs $T(x)$ and $T(y)$.