Mathematics of Life Contingencies. Math 3280 3.00 F

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Homework 5

Unless otherwise indicated, all lives in the following questions are subject to the same law of mortality and their times until death are independent random variables.

1. Under the assuption of uniform distribution of deaths, show that

   a. $\overset{\circ}{e}_x = e_x + \frac{1}{2}$

   b. $\mathbf{Var}(T) = \mathbf{Var}(K) + \frac{1}{12}$

2. Consider a modification of De Moivre's law given by

$$s(x) = (1 - \frac{x}{w})^{\alpha} \qquad 0 \leq x < w, \qquad \alpha > 0$$

   Calculate

   a. $\mu(x)$ *b.* $\overset{\circ}{e}_x$

3. Using life table and an assumption of uniform distribution of deaths in each year of age to find the median of the future lifetime of a person

   a. Age 0   b. Age 50

4. The recursion formula is to be used to produce tables of compound interest functions. Find u(1), -c(x)/d(x), and 1/d(x) when

   a. $u(x) = \ddot{a}_{\overline{n}|}$

   b. $u(x) = \ddot{s}_{\overline{n}|}$

5. Graph $\mu(x + t) \quad 0 < t < 1$, for each of the three assumptions. Also graph the survival function for each assumption.

6. A population has 30% who are smokers with a constant force of mortality 0.2 and 70% who are non-smokers with a constant force of mortality 0.1.

   Calculate the 75th percentile of the distribution of the future lifetime of an individual selected at random from this population.

7. For T, the future lifetime random variable for (0):

   a. $w > 70$

   b. ${}_{40}p_0 = 0.6$

   c. $\mathbf{E}[T] = 60$

   d. $\mathbf{E}[min(T,t)] = t - 0.005t^2, \quad 0 < t < 60$

   Calculate the complete expectation of life at 40.

8. You are given that $e_{39} = 49$ and $p_{35} = 0.995$.

   If μ_x is doubled for $35 \leq x \leq 36$, what is the revised value of e_{35} ?

9. For a life age 50, the curtate expectation of life $e_{50} = 20$. For that same life, you are also given that $p_{50} = 0.97$.

   Determine e_{51}.

10. You are given :

    1) $\mathring{e}_{40} = 75$.

    2) $\mathring{e}_{60} = 70$.

    3) The force of mortality μ_x for x$\in [40, 60]$ is 1/(k-x) for some k.

    Determine k.

GOOD LUCK!