Mathematics of Life Contingencies. Math 3280 3.00 F

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Homework 4—–solution

Unless otherwise indicated, all lives in the following questions are subject to the same law of mortality and their times until death are independent random variables.

1. You are given:

   a) $d_{48} = 80$

   b) $l_{50} = 450$

   c) ${}_{3|2}q_{45} = 1/6$

   d) ${}_{3}p_{45} = 2/3$

   Determine d_{49}.

   solution

   Since ${}_{3|2}q_{45} ={}_3 p_{45}\ {}_2q_{48}$, we deduce ${}_2q_{48} = (1/6)/(2/3) = 1/4$, and therefore $l_{50}/l_{48} = 1-\ {}_2q_{48} = 3/4$ and $l_{48} = 450(4/3) = 600$. Then $l_{50} = l_{48} - d_{48} - d_{49}$, or $450 = 600 - 80 - d_{49}$, implying $d_{49} = 70$.

2. You are given:

   a) The probability that a person age 50 is alive at age 55 is 0.9.

   b) The probability that a person age 55 is not alive at age 60 is 0.15.

   c) The probability that a person age 50 is alive at age 65 is 0.54.

   Calculate the probability that a person age 55 dies between ages 60 and 65.

   solution

   We need ${}_{5|5}q_{55} ={}_5 p_{55} -{}_{10} p55$.

$${}_5p_{55} = 1 -{}_5 q_{55} = 1 - 0.15 = 0.85$$

$$
{}_{15}p_{50} = 0.6
$$

$$
{}_{5}p_{50}\ {}_{10}p_{55} = 0.54
$$

$$
0.9\,{}_{10}p_{55} = 0.54
$$

$$
{}_{10}p_{55} = 0.6
$$

3. You are given the following mortality table:

x	l_x	q_x	d_x
50	1000	0.02	
51			32
52			30
53			28
54		0.028	

   In a group of 800 people age 50, determine the expected number who will die while age 54.

   solution

   $l_{54} = 1000(1-0.02) - 32 - 30 - 28 = 890$. Then ${}_{4|}q_{50} = (890/1000)(0.028) = 0.02492$. For 800 people, $800(0.02492) = 19.936$.

4. For a population of individuals, you are given:

   a) Each individual has a constant force of mortality.

   b) The forces of mortality are uniformly distributed over the interval (0,2).

   Calculate the probability that an individual drawn at random from this population dies within one year.

   solution

   The law of total probability says that $\Pr(x) = \int \Pr(X|y) f(y) dy$, where

$$
\int_0^2 \frac{1}{2}(1 - e^{-\mu}) d\mu = \frac{1}{2}(2 + e^{-2} - 1) = 0.5677
$$

5. You are given:

a) $\mu_{35+t} = \mu$, $0 \leq t \leq 1$.

b) $p_{35} = 0.985$.

c) μ^*_{35} is the force of mortality for (35) subject to an additional hazard, $0 \leq t \leq 1$.

d) $\mu^*_{35} = \mu + c, \quad 0 \leq t \leq 0.5$.

e) The additional force of mortality decreases uniformly from c to 0 between age 35.5 and 36.

Determine the probability that (35) subject to the additional hazard will not survive to age 36.

solution

Let p^*_{35} be the modified probability of survival. Then,

$$p^*_{35} = e^{-\int_0^1 \mu^*_{35+t} dt} = e^{-\int_0^1 \mu_{35+t} dt} \; e^{-(\int_0^1 \mu^*_{35+t} - \mu_{35+t} dt)} = p_{35} \; e^{-(\int_0^1 \mu^*_{35+t} - \mu_{35+t} dt)}$$

So $P^*_x = p_x$ times e raised to the integral of negative the additional force of mortality from 0 to 1. The additional force of mortality is the difference between the two lines. The area is 0.5(0.5+1)c = 0.75c. So $p^* = 0.985e^{-0.75c}$.

6. You are given:

a) $q_x = 0.1$.

b) Uniform distribution of deaths between integral ages is assumed.

Calculate ${}_{1/2}q_{x+(1/4)}$.

solution

Let $l_x = 1$. Then $l_{x+1} = l_x(1 - q_x) = 0.9$ and $d_x = 0.1$. Linearly interpolating,

$$l_{x+1/4} = l_x - \frac{1}{4} d_x = 0.975$$
$$l_{x+3/4} = l_x - \frac{3}{4} d_x = 0.925$$

$$ {}_{1/2}q_{x+1/4} = \frac{l_{x+3/4} + l_{x+1/4}}{l_{x+1/4}} = 0.051282 $$

7. You are given:

   a) ${}_{0.25}q_{x+0.75} = 3/31$.

   b) Mortality is uniformly distributed within age x.

   Calculate q_x.

   solution

$$ \frac{3}{31} ={}_{0.25} q_{x+0.75} = \frac{0.25q_x}{1-0.75q_x} $$
$$ \frac{3}{31} - \frac{2.25}{31}q_x = 0.25q_x $$
$$ \frac{3}{31} = \frac{10}{31}q_x $$
$$ q_x = 0.3 $$

8. A mortality study is conducted for the age interval (x, x+1].

   If a constant force of mortality applies over the interval, ${}_{0.25}q_{x+0.1} = 0.05$.

   Calculate ${}_{0.25}q_{x+0.1}$ assuming a uniform distribution of deaths applies over the interval.

   solution

   Under constant force, ${}_{s}p_{x+t} = p_x^s$, so $p_x = 0.95^4 = 0.814506$, $q_x = 1 - 0.814506 = 0.185494$. Then under a uniform assumption

$$ {}_{0.25}q_{x+0.1} = \frac{0.25q_x}{1-0.1q_x} = \frac{(0.25)(0.185494)}{1-0.1(0.185494)} = 0.04725 $$

9. If the UDD assumption is valid for (x), does UDD hold for $\overset{1}{x}:\overline{n|}$?

10. If the UDD assumption is valid for (x), does UDD hold for $x:\overset{1}{\overline{n|}}$?

11. If the UDD assumption is valid for (x), does UDD hold for $x:\overline{n|}$?

12. If the UDD assumption is valid for each of (x) and (y) and if (x) and (y) are independent lives, does UDD hold for x : y?

GOOD LUCK!