

Question 1 Use ILT and the assumption of UDD to compute ${}_{2.25}p_{85.5}$.

Question 2 Consider the joint life status $(x : y)$, where $x \geq 0$, $y \geq 0$ and the RV's $T(x)$ and $T(y)$ are independent and identically distributed as a uniform RV $U \sim \text{Uni}[0, 1]$. Compute the full expectancy of life ${}^{\circ}e_{x:y}$.

Q. 1

$$\begin{aligned}
 {}_{2.25}p_{85.5} &= 0.5 p_{85.5} \cdot p_{86} \cdot 0.75 p_{87} \\
 &= \frac{l_{86}}{l_{85.5}} \cdot \frac{l_{87}}{l_{86}} \cdot \frac{l_{87.75}}{l_{87}} \\
 &= \frac{l_{87.75}}{l_{85.5}} = \frac{0.75 l_{88} + 0.25 l_{87}}{0.5 l_{86} + 0.5 l_{85}} \\
 &= \frac{0.75 \cdot 15,247.58 + 0.25 \cdot 19,872.99}{0.5 \cdot 20,660.90 + 0.5 \cdot 23,582.45} \\
 &\approx \frac{15,904}{22,122} \approx 0.7189
 \end{aligned}$$

Q. 2

$$\begin{aligned}
 {}^{\circ}e_{x:y} &\stackrel{U}{=} \int_0^1 t p_x \cdot t p_y dt = \int_0^1 (1-t)(1-t) dt = \int_0^1 (1-t)^2 dt \\
 &= \int_0^1 \frac{1}{3} d(1-t)^3 = \frac{1}{3}
 \end{aligned}$$