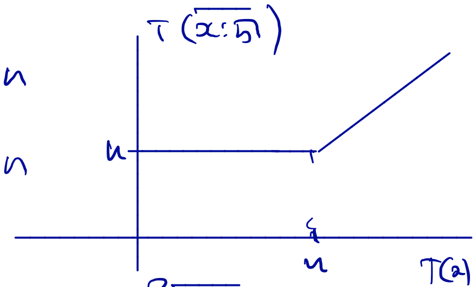
Question 1 Recall that the last survivor life status $(\overline{x:\overline{n}|})$, $x \geq 0$, $n \in \mathbb{N}$ dies upon the latest of the deaths of the two life statuses (x) and $(\overline{n}|)$. Assume that the UDD assumption (see the list of useful expressions) holds for the life status (x), does this imply that the UDD approximation holds for the last survivor life status, too? Prove or disprove.

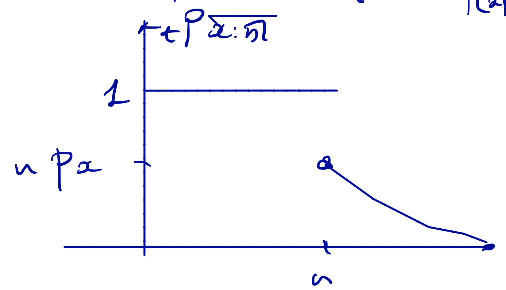
Question 2 Assume that the random variable (RV) $T(x)$, $x \geq 0$ is distributed exponentially with the rate parameter $\mu \in \mathbb{R}_+$. What is the distribution of the RV $T(x+t)$, $t \in \mathbb{R}_+$? What is $\mathring{e}_{x+t}$?

Q 1

Recall that $T(\overline{x:\overline{n}|}) = \begin{cases} T(x), & T(x) \geq n \\ n, & T(x) < n \end{cases}$



Hence ${}_tp_{\overline{x:\overline{n}|}} = \begin{cases} 1, & t < n \\ {}_tp_x, & t \geq n \end{cases}$



As UDD holds on (x), we have

${}_{k+t}p_x \approx (1-t)\,{}_{k+1}p_x + t\,{}_kp_x$, $k \in \mathbb{N}$, $t \in [0,1)$

Assume that UDD holds on $(\overline{x:\overline{n}|})$ and choose $k = n-1$. Then

${}_{n-1+t}p_{\overline{x:\overline{n}|}} = 1$

whereas

${}_np_{\overline{x:\overline{n}|}} = {}_np_x$ and ${}_{n-1}p_{\overline{x:\overline{n}|}} = 1$

Hence we arrive at

${}_{k+t}p_{\overline{x:\overline{n}|}} \neq (1-t)\,{}_{k+1}p_{\overline{x:\overline{n}|}} + t\,{}_kp_{\overline{x:\overline{n}|}}$ for $k = n-1$, thus a contradiction. Hence UDD does not hold on $(\overline{x:\overline{n}|})$.

Q, 2

$\mathbb{P}(T(x+t) \geq u) = \mathbb{P}(T(x) - t \geq u \mid T(x) \geq t) = \mathbb{P}(T(x) \geq u+t \mid T(x) \geq t)$
$= \mathbb{P}(T(x) \geq u+t) / \mathbb{P}(T(x) \geq t) = e^{-\mu(u+t)} / e^{-\mu t} = e^{-\mu u}$, $u \in [0, \infty)$

Hence $T(x+t) \sim Exp(\mu)$. Then $\mathring{e}_{x+t} = \mathring{e}_x = \mu^{-1}$.

The End.