Mathematics of Life Contingencies. Math 3280 3.00 F

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Homework 3

Unless otherwise indicated, all lives in the following questions are subject to the same law of mortality and their times until death are independent random variables.

1. If $s(x) = 1 - \frac{x}{100}$, $0 \le x \le 100$, calculate:

   a. $\mu(x)$

   b. $F_X(x)$

   c. $f_X(x)$

   d. $\Pr(10 < X < 40)$.

2. Given the survival function of question 1, determine the survival function, force of mortality, and p.d.f of the future lifetime of (40).

3. If $\mu(x) = 0.001$ for $20 \le x \le 25$, evaluate ${}_{2|2}q_{20}$.

4. Show that

$$\frac{d}{dx}\,{}_tp_x = {}_tp_x\,[\mu(x) - \mu(x+t)]$$

5. If $\mu(x+t) =$ t, $t \ge 0$, calculate ${}_tp_x\ \mu(x+t)$.

6. You are given that

$$s(x) = (\frac{100}{100+x})^2$$

   Calculate ${}_5q_{40}$.

7. You are given $\mu_x = \frac{a}{w-x}$, prove that

$$ {}_tp_x = (\frac{w-x-t}{w-x})^a$$

8. You are given that the force of mortality is

$$\mu_x = \frac{0.5}{100 - x}$$

Calculate the probability that (36) survives to age 75.

9. You are given:

1) $\hat{\mu}_{x+t} = \mu_{x+t} - k,\ 0 \leq t \leq 1$

2) $\hat{q}_x = 0$ where $\hat{q}_x$ is based on the force of mortality $\hat{\mu}_{x+t}$

Determine k.

GOOD LUCK!