

① We need $\mathbb{P}[M|N=2]$ and so $\mathbb{P}(M=m|N=2)$, $m \in \mathbb{N}$. To this end

$$\begin{aligned} \mathbb{P}(N=2) &= \sum_{m=1}^{\infty} \mathbb{P}(M=m, N=2) = \sum_{m=1}^{\infty} \frac{3}{16} e^{-2} (1-e^{-2})^{m-1} \\ &= \frac{3}{16} e^{-2} \sum_{m=0}^{\infty} (1-e^{-2})^m = \frac{3e^{-2}}{16} \frac{1}{e^{-2}} = \frac{3}{16}. \end{aligned}$$

Then

$$\mathbb{P}(M=m|N=2) = \frac{\mathbb{P}(M=m, N=2)}{\mathbb{P}(N=2)} = \frac{16}{3} \cdot \frac{3}{16} e^{-2} (1-e^{-2})^{m-1}$$

$= e^{-2} (1-e^{-2})^{m-1}$, $m \in \mathbb{N}$.
You can thus say that $M|N=2 \sim \text{Geom}(e^{-2})$, so $\mathbb{P}[M|N=2] = e^{-2}$.

Otherwise:

$$\begin{aligned} \mathbb{P}[M|N=2] &= \sum_{m=1}^{\infty} m e^{-2} (1-e^{-2})^{m-1} = p \sum_{m=0}^{\infty} (1-p)^{m-1} \\ &= p \sum_{m=0}^{\infty} \frac{d}{dp} (1-p)^m = -p \frac{d}{dp} \sum_{m=0}^{\infty} (1-p)^m = -p \frac{d}{dp} \frac{1}{p} = \frac{1}{p}. \end{aligned}$$

② The density of the RV, T , is $f(t) = \frac{1}{3} e^{-\frac{1}{3}t}$, $t \in \mathbb{R}_+$.
Also:

$$T \vee 2 = \begin{cases} T, & T > 2 \\ 2, & T \leq 2 \end{cases}$$

$$\begin{aligned} \text{So } \mathbb{P}[T \vee 2] &= \mathbb{P}[T \vee 2 > 2] + 2 \mathbb{P}[T \vee 2 \leq 2] \\ &= 2(1 - e^{-\frac{2}{3}}) + e^{-\frac{2}{3}} \mathbb{P}[T|T > 2] \\ &= 2(1 - e^{-\frac{2}{3}}) + e^{-\frac{2}{3}} (\mathbb{P}[T-2|T > 2] + 2) \\ &= 2(1 - e^{-\frac{2}{3}}) + e^{-\frac{2}{3}} \cdot (3 + 2) \\ &= 2 + 3e^{-\frac{2}{3}}. \end{aligned}$$